CHAPTER 6
Lines and Angles

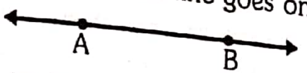
6.1 Introduction

Point

A point is represented by a dot. It has no dimensions like length, breadth or thickness. It has only position. Points are denoted by capital letters A, B, C, D etc.
A dot made by sharp pencil is a point.

Line

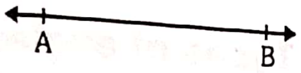
A geometrical line is a set of points that extends endlessly in both the directions i.e., a line has no end points. It has only length. The arrow heads show that the line goes on endlessly on either side.



Lines are denoted by small letters ℓ, m, n,.....

Line segment

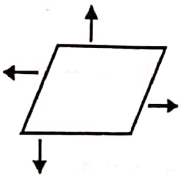
A line segment is a line which has end points. In the figure, the part of the line between the points 'A' and 'B', including 'A' and 'B' is a line-segment.



The line segment AB is represented as $\overline{AB}$ or segment AB.

Plane

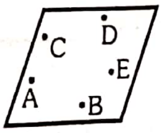
A plane is a set of points. It is a flat surface with length and breadth. A geometrical plane extends endlessly in all the directions. Small letters are used to denote a plane.



e.g.,Surface of a sheet of paper, surface of a wall.

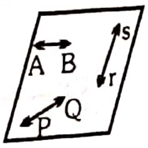
Coplanar points

The points that belong to the same plane are called coplanar points.



Coplanar lines

The lines that lie in the same plane are called coplanar lines.



Space

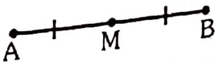
The universal set of points, lines and planes is called a space. It has no end.

Distance between two points

The distance between two points A and B is the length of the line segment joining them. The distance between A and B is denoted by d(A, B) or AB.

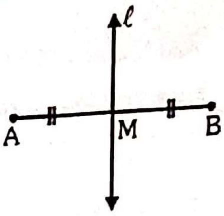
Midpoint

Given a line segment AB, a point M is said to be the midpoint of AB, if M is an interior point of AB, such that AM = MB.



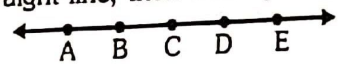
Perpendicular bisector

A line 'ℓ' passing through the mid point 'M' of a line segment $\overline{AB}$ and perpendicular to AB is called the perpendicular bisector of the line segment $\overline{AB}$.



Collinear points

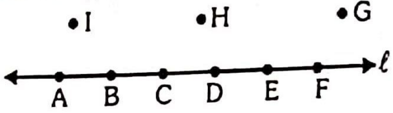
If three or more points lie on a straight line, then those points are called collinear points.



A, B, C, D, E are collinear.

Non-collinear points

The points which do not lie on the straight line are called non-collinear points.

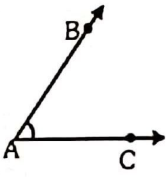


G, H, I do not lie on the straight line 'ℓ'.
Hence, they are non-collinear points.

6.2 Types of angles

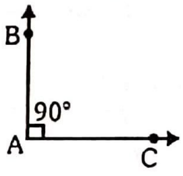
Acute angle

An angle which is less than 90° is called an acute angle.



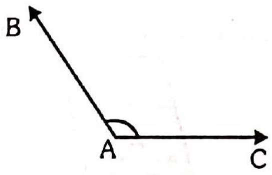
Right angle

An angle which is equal to 90° is called a right angle.



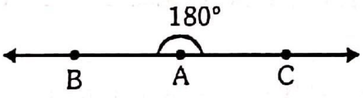
Obtuse angle

An angle which is greater than 90° but less than 180° is called an obtuse angle.



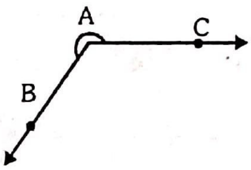
Straight angle

An angle which is equal to 180° is called a straight angle.



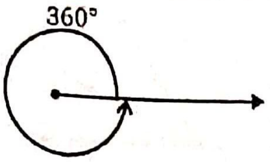
Reflex angle

An angle which is greater than 180° but less than 360° is called reflex angle.



Complete angle

An angle which is exactly equal to 360° is called a complete angle.



Zero angle

An angle is said to be a zero angle when two rays coincide and the measure of angle is 0°.

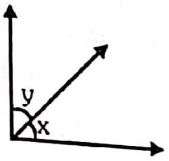
Pair of angles

Complementary angles

When the sum of the measure of two angles is equal to 90°, then the angles are said to be complementary angles.

$\angle x + \angle y = 90°$

$\therefore$ $\angle x$ and $\angle y$ are complementary angles.

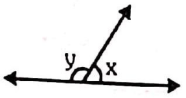


Supplementary angles

When the sum of the measure of two angles is equal to 180°, then the angles are said to be supplementary angles.

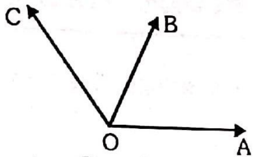
$\angle x + \angle y = 180°$

$\therefore$ $\angle x$ and $\angle y$ are supplementary.



Adjacent angles

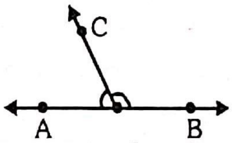
Angles having the same vertex and a common side and which lie on the opposite sides of the common side are called adjacent angles.



Angles AOB and COB with common vertex O and common side OB are adjacent angles.

Linear pair of angles

Two adjacent angles are said to form a linear pair of angles, if they lie on the same straight line. The two non-common arms are opposite rays.



The sum of two adjacent angles of a linear pair of angles is 180°, hence they are also supplementary.

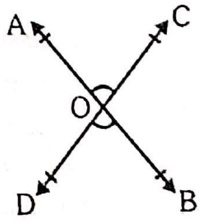
Congruent angles

Two angles are said to be congruent if they have the same measure.

Vertically opposite angles

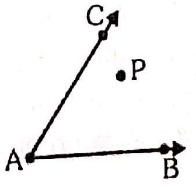
If two lines AB and CD intersect at a point 'O', then the pair of angles ∠AOC and ∠BOD is said to be a pair of vertically opposite angles. Also ∠AOD and ∠BOC form another pair of vertically opposite angles.

Angles forming a pair of vertically opposite angles are congruent (or equal).



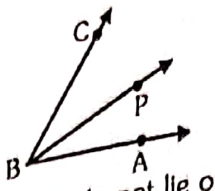
Interior of an angle

It is a set of points on the same side of the lines forming the angles.

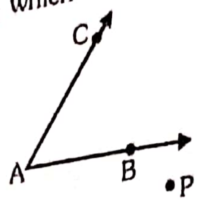


Angle addition axiom

m∠ABC = m∠ABP + m∠PBC



Exterior of an angle is the set of points, which do not lie on the angle or in its interior.



The sum of the measures of the angles formed around a point is 360°.

Angles in a transversal

(i) Interior angles are the angles inside the parallel lines and on both sides of the transversal.

(ii) Co-interior angles are the interior angles lying on the same side of the transversal.

(iii) Exterior angles are the angles lying outside the parallel lines and on both sides of the transversal.

(iv) Corresponding angles are the angles on the same side of the transversal with one angle interior and the other angle exterior and the angles are not adjacent angles.

(v) If a transversal intersects two coplanar parallel lines, then the corresponding angles are equal. This is corresponding angles axiom.

(vi) Alternate angles are the interior angles lying on either side of the transversal and are not adjacent angles.

6.3 Properties of angles

(i) The adjacent angles formed when one straight line stands over another are together equal to two right angles i.e., 180°.

(ii) If two adjacent angles are supplementary then their outer arms are on a straight line.

(iii) If two straight lines cut one another, the four angles so formed are together equal to four right angles i.e., 360°.

(iv) If two straight lines cut one another, the vertically opposite angles are equal.

(v) When a number of straight lines meet at a point the sum of all angles so formed at that point is equal to four right angles i.e., 360°.

6.4 Properties of parallel lines

(i) If a transversal intersects two coplanar parallel lines in such a way that

  (a) A pair of alternate angles are equal then the two lines are parallel.

  (b) A pair of interior angles on the same side of the transversal are supplementary, then the two lines are parallel.

(ii) If a transversal intersects a pair of parallel lines, then

  (a) The interior angles on the same side of the transversal are supplementary.

  (b) Each pair of alternate interior angles are equal.

(iii) Straight lines which are parallel to the same straight line are themselves parallel to one another.

(iv) If there are three or more parallel straight lines and the intercepts made by them on any transversal are equal, then the corresponding intercepts on any other transversal are also equal.

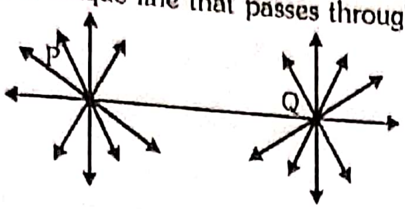
6.5 Euclid's postulates

Postulate-1

A straight line may be drawn from any one point to any other point.

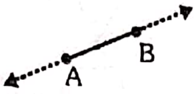
Axiom

Given two distinct points, there is a unique line that passes through them.



Postulate-2

A terminated line can be produced indefinitely.



Postulate-3

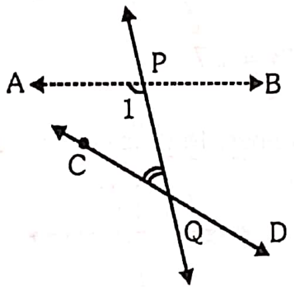
A circle can be drawn with any centre and of any radius.

Postulate-4

All right angles are equal to one another.

Postulate-5

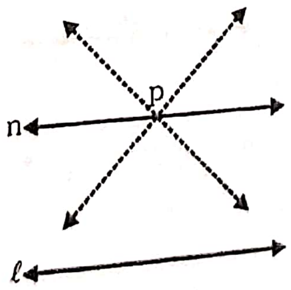
If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then two straight lines, if produced indefinitely, meet that side on which the angles are less than two right angles.



6.6 Theorems

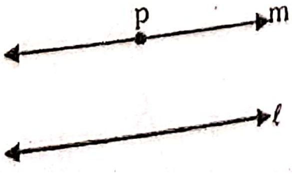
Theorem-1 (playfair's axiom)

For every line 'ℓ' and for every point 'p' not lying on 'ℓ', there exists a unique line 'n' passing through 'p' and parallel to 'ℓ'.



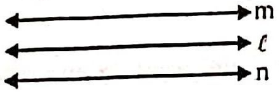
Theorem-2 (parallel axiom)

If 'ℓ' is a line and 'p' is a point not on 'ℓ' then there is one and only one line which passes through 'p' and is parallel to 'ℓ'.



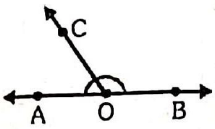
Theorem-3

Two lines which are both parallel to the same line are parallel to each other.



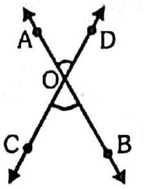
Theorem-4 (linear pair axiom)

If a ray stands on a line, then the sum of two adjacent angles so formed is 180°.



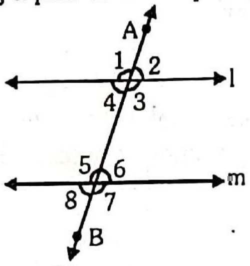
Theorem-5

If two lines intersect, then the vertically opposite angles are equal.



Theorem-6 (corresponding angles)

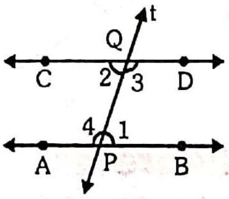
If a transversal intersects two parallel lines, then each pair of corresponding angles are equal, conversely if a transversal intersects two lines, making a pair of corresponding angles are equal, then the lines are parallel.



$\angle 1 = \angle 5$, $\angle 4 = \angle 8$, $\angle 2 = \angle 6$ & $\angle 3 = \angle 7$

Theorem-7

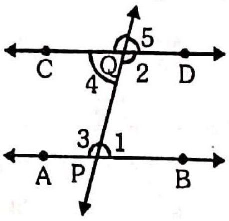
If a transversal intersects two parallel lines, then each pair of alternate angles are equal.



$\angle 2 = \angle 1$, $\angle 3 = \angle 4$

Theorem-8

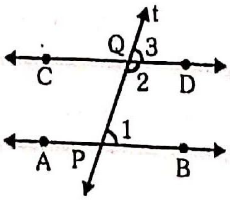
If a transversal intersects two parallel lines, then each pair of consecutive interior angles are supplementary.



$\angle 2 + \angle 1 = 180°$ & $\angle 4 + \angle 3 = 180°$

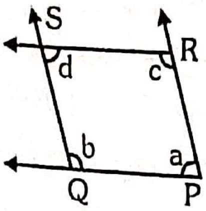
Theorem-9

If a transversal intersects two lines in such a way that a pair of consecutive interior angles are supplementary, then the two lines are parallel.



EXERCISE

Multiple choice questions

1.


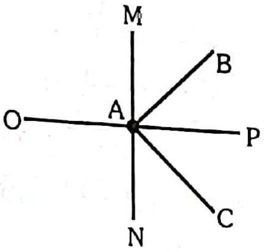
In the above figure (not to scale), $\overrightarrow{PQ} \parallel \overrightarrow{RS}$ and $\overrightarrow{PR} \parallel \overrightarrow{QS}$. Each pair of parallel rays is intersecting the other pair and a, b, c and d are the angles formed. If c = 120°, then a is

(1) 80°
(2) 70°
(3) 60°
(4) 90°

2. If x° is the measure of an angle which is equal to its complement and y° is the measure of the angle which is equal to its supplement, then x°/y° is

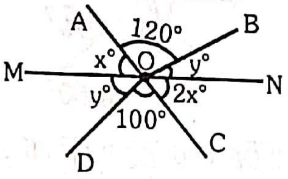
(1) 1
(2) 3
(3) 0.5
(4) 2

3. In the figure given below (not to scale), MN and OP intersect at right angles. If (∠MAB : ∠BAP) = 2 : 1 = (∠PAC : ∠CAN), then (∠MAB + ∠PAC) is



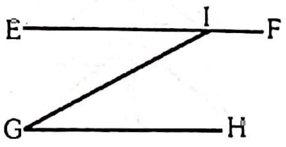
(1) 120°
(2) 90°
(3) 30°
(4) 60°

4. In the figure below (not to scale), MN is a straight line. ∠AOB = 120°, ∠DOC = 100°, the relation between x° and y° is



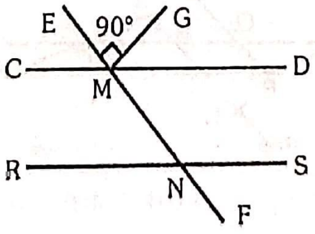
(1) x° = 2y°
(2) y° = 2x°
(3) y° = x°
(4) x° = 3y°

5. In the figure below, $\overline{EF}$ and $\overline{GH}$ are parallel to each other. If $\overline{GI}$ is the transversal, ∠IGH = y° and ∠FIG = 3y° then the ratio of the supplement and the complement of y° is



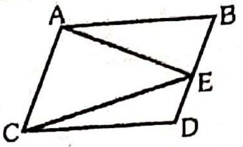
(1) 2 : 1
(2) 3 : 1
(3) 4 : 3
(4) 3 : 2

6. If the figure below, $\overline{CD}$ is parallel to $\overline{RS}$. ∠EMG = 90°, ∠GMD = y°, ∠CME = x° and $y° = \frac{x°}{2}$. Find ∠FNS : ∠FNR.



(1) 1 : 2
(2) 2 : 1
(3) 1 : 1
(4) 3 : 2

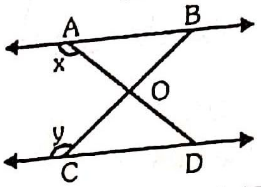
7.



In the above figure (not to scale), $\overline{AB} \parallel \overline{CD}$. If ∠BAE = 25° and ∠DCE = 30°, then find ∠AEC.

(1) 30° (2) 45° (3) 50° (4) 55°

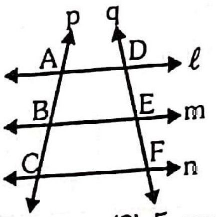
8.



In the above figure (not to scale), x = 125°, y = 135° and $\overline{AB} \parallel \overline{CD}$. Find ∠AOB.

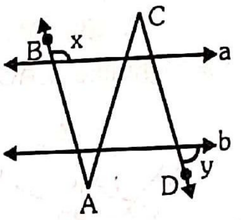
(1) 40° (2) 60° (3) 80° (4) 100°

9. In the figure below (not to scale), $\ell \parallel m \parallel n$, p and q are transversals, AB : BC = 2 : 1 and DE = 5 cm. Find EF.



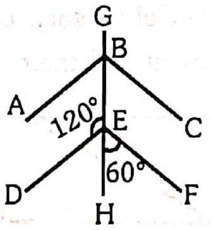
(1) 3 cm (2) 2 cm (3) 5 cm (4) 2.5 cm

10. In the figure below (not to scale), a ∥ b, and x = 130° $\overline{AB} \parallel \overline{CD}$. Find the difference between x and y.



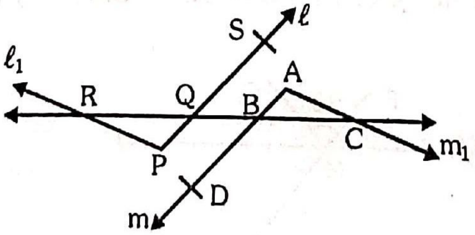
(1) 80° (2) 70° (3) 60° (4) Data inadequate

11. In the figure below (not to scale), $\overline{AB} \parallel \overline{DE}$ and $\overline{BC} \parallel \overline{EF}$. If ∠HEF = 60° and ∠DEB = 120°, then find the ratio of ∠ABG and ∠EBC.



(1) 1 : 1 (2) 2 : 1 (3) 3 : 2 (4) 5 : 4

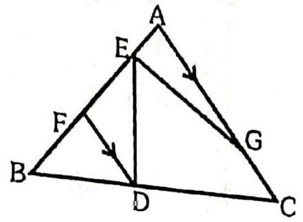
12.



In the above figure, $\ell \parallel m$ and $\ell_1 \parallel m_1$. If ∠ACB = 40° and ∠DBC = 110°, then find ∠QPR.

(1) 60° (2) 70° (3) 80° (4) 110°

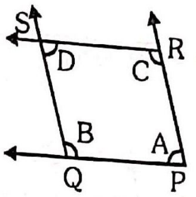
13.



In the above ΔABC, $\overline{AC} \parallel \overline{DE}$ and $\overline{DF} \parallel \overline{EG}$. If $\angle FDE = 65°$ then find $\angle EGA$.

(1) 65°
(2) 70°
(3) 115°
(4) 150°

14. In the below figure $\overrightarrow{PQ} \parallel \overrightarrow{RS}$ and $\overrightarrow{PR} \parallel \overrightarrow{QS}$. Each pair of parallel rays is intersecting the other pair. $\angle A$, $\angle B$, $\angle C$ and $\angle D$, are the angles formed. If $\angle C = 110°$, then $\angle B$ =

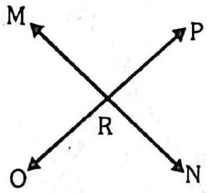


(1) 90°
(2) 110°
(3) 100°
(4) 120°

15. The ratio between the complement and the supplement of an angle is 1 : 2. The angle is

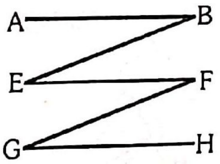
(1) 90°
(2) 180°
(3) 135°
(4) 0°

16. MN and OP are two straight lines intersecting at R. If $\angle ORN = 5/2\ (\angle ORM)$, find $\angle NRP : \angle MRP$.



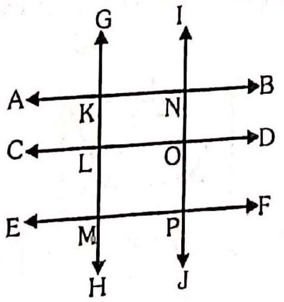
(1) 2 : 5
(2) 5 : 2
(3) 1 : 1
(4) 3 : 7

17. AB, EF, and GH are parallel lines, also EB and GF are parallel to each other. If the supplement of the $\angle ABE$ is 151.5° then the complement of the $\angle FGH$ is



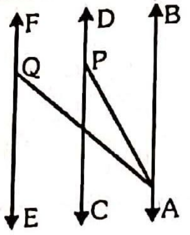
(1) 61.5°
(2) 79.8°
(3) 90°
(4) 28.5°

18. Both AB and CD are parallel to EF and are perpendicular to GH. GH and IJ are parallel to each other. If KL : LM = 2 : 3 then ON : OP =



(1) 2 : 3
(2) 3 : 2
(3) 2 : 5
(4) 3 : 5

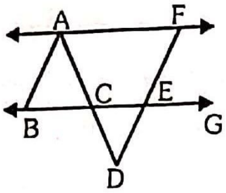
19.



In the above figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$, ∠DPA = 130°, and ∠AQE = 80°. Find ∠PAQ.

(1) 30° (2) 25° (3) 60° (4) 80°

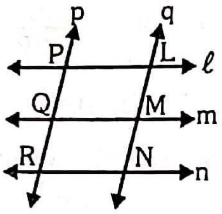
20.



In the above figure, AF ∥ BE and AB ∥ FE. ∠AFE = 40° and ∠BAC = 80°. Find ∠DCE.

(1) 120° (2) 100° (3) 80° (4) 60°

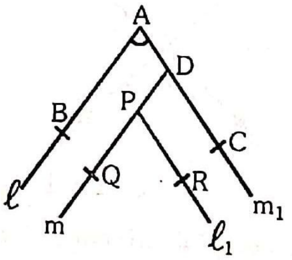
21.



In the above given figure, ℓ∥m∥n. PQ : QR = 4 : 3 and LN = 21 cm. Find the lengths of LM and MN (in cm) respectively.

(1) 9 cm, 15 cm (2) 9cm, 12cm (3) 12 cm, 9 cm (4) 15 cm, 9 cm

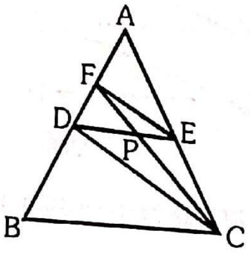
22.



In the above figure if ℓ ∥ m and $\ell_1 \parallel m_1$. If ∠BAC = 55° then find ∠QPR.

(1) 60° (2) 75° (3) 55° (4) 125°

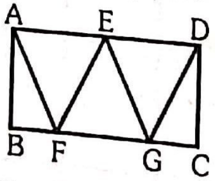
23.



In the figure above (not to scale), $\overline{DE} \parallel \overline{BC}$, $\overline{EF} \parallel \overline{DC}$, ∠EFC = 30° and ∠FED = 40°. Find ∠BCF.

(1) 45° (2) 50° (3) 60° (4) 70°

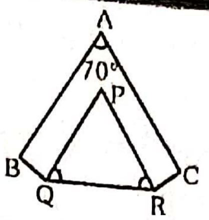
24.



In the figure above (not to scale), $\overline{EF} \parallel \overline{GD}$, $\overline{AF} \parallel \overline{EG}$, $\overline{AD} \parallel \overline{BC}$ and ∠DCG = 100°. If ∠CDG = 40°, then find ∠AEF.

(1) 30° (2) 40° (3) 150° (4) 60°

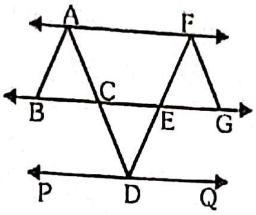
25.



In the figure above (not to scale), $\overline{AB}\parallel\overline{PQ}$ and $\overline{AC}\parallel\overline{PR}$. If ∠BAC = 70° and ∠PQR = 50° find ∠PRQ.

(1) 30° (2) 50° (3) 60° (4) 80°

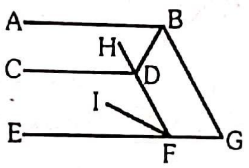
26.



In the figure above (not to scale), $\overline{AF}\parallel\overline{BG}\parallel\overline{PQ}$, $\overline{AB}\parallel\overline{EF}$ and $\overline{AD}\parallel\overline{FG}$. ∠ABC = 30° and ∠FGE = 70°. Find ∠CDE.

(1) 20° (2) 40° (3) 60° (4) 80°

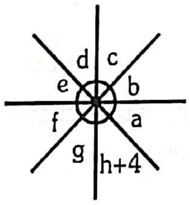
27.



In the figure above (not to scale), EFG is a straight segment. ∠ABD = 60°, ∠HDC = 60°, ∠BGE = 80° and ∠IFE = 30°. If $\overline{DH}$ bisects ∠BDC and $\overline{FI}$ bisects ∠DFE then find DBG.

(1) 30° (2) 40° (3) 50° (4) 60°

28.



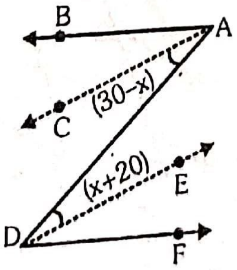
In the figure above (not to scale), the values of angles a, b, c, d, e, f, g and h are consecutive integers in increasing order. Find the value of a.

(1) 38 (2) 39 (3) 40 (4) 41

29. There are two angles a and b such that a = b + 15°. If the supplement of 3b is twice the complement of b + 15° find the value of a.

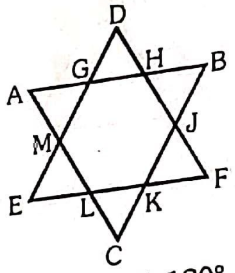
(1) 30 (2) 45 (3) 50 (4) 55

30. In the figure below (not to scale), $\overline{AC}$ and $\overline{DE}$ are bisectors of ∠BAD and ∠ADF respectively. If $\overline{AC}\parallel\overline{DE}$, then find the measure ∠BAD + ∠ADF.



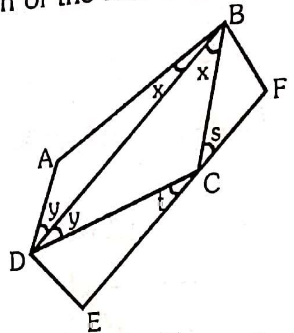
(1) 100° (2) 80° (3) 50° (4) None of the above

31. In the given figure, $\overline{AB} \parallel \overline{EF}$, $\overline{DE} \parallel \overline{BC}$ and $\overline{AC} \parallel \overline{DF}$. If ∠EDF = 70°, then find ∠KLC + ∠LKC.



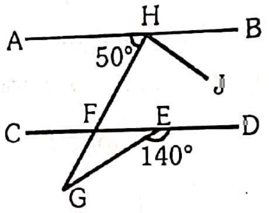
(1) 70° (2) 110° (3) 120° (4) 80°

32. In the figure below (not to scale), $\overline{AB} \parallel \overline{CD}$ and $\overline{BD} \parallel \overline{EF}$. If ∠ABD = ∠DBC = x, ∠BDC = ∠ADB = y, ∠BCF = s and ∠DCE = t, then which of the following is correct?



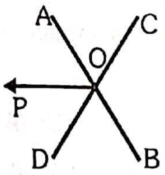
(1) x = t and y = s (2) x = s and y = t (3) x = y = t = s (4) None of the above

33. In the figure below (not to scale), $\overline{AB} \parallel \overline{CD}$, ∠AHF = 50° and ∠DEG = 140°. If ∠BHJ = 2∠HGE then find ∠BHJ.



(1) 10° (2) 20° (3) 30° (4) 40°

34. In the figure below (not to scale), $\overline{AB}$ and $\overline{CD}$ intersect at the point O. ∠AOC = 100° and $\overrightarrow{OP}$ is the bisector of ∠AOD. Find the measure of reflex ∠POB.

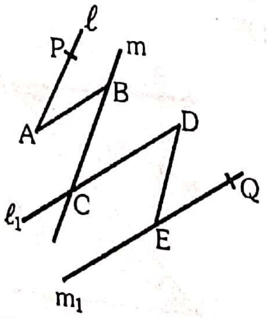


(1) 200° (2) 190° (3) 210° (4) 220°

35. If the difference between an angle and its supplement is 100°, then find the ratio of the larger and the smalle angles.

(1) 3 : 2 (2) 7 : 1 (3) Both (1) and (2) (4) None of the above

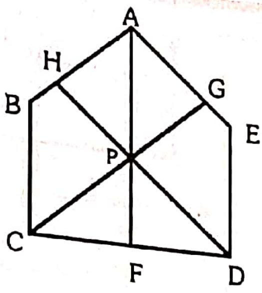
36.



In the above figure, ℓ ∥ m, $\ell_1 \parallel m_1$, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DE}$. If ∠PAB = 28°. Then find ∠DEQ.

(1) 28° (2) 56° (3) 84° (4) 52°

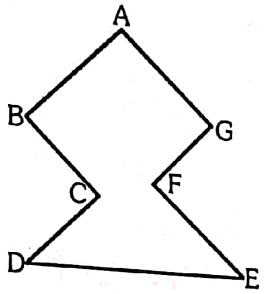
37.



In the above figure, $\overline{AF} \parallel \overline{ED}$, $\overline{CG} \parallel \overline{AB}$ and $\overline{AE} \parallel \overline{HD}$. If $\angle FPD = 40°$. Then find $\angle AED$.

(1) 40°
(2) 80°
(3) 120°
(4) 140°

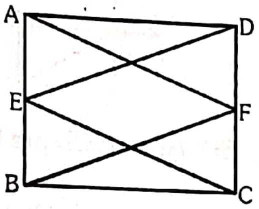
38.



In the above figure, EF||AG, AB||CD||FG and AG||BC. If $\angle EFG = 70°$ then find $\angle BAG - \angle BCD$.

(1) 70°
(2) 40°
(3) 80°
(4) 110°

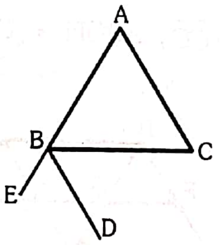
39.



In the above figure (not to scale), E and F are the mid points of AB and CD respectively. AB||CD, BC||AD and $\angle ADE = 70°$, and $\angle BCE = 40°$. $\angle DEC$ is

(1) 70°
(2) 40°
(3) 110°
(4) 120°

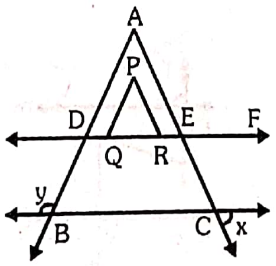
40.



In the figure above (not to scale), $\overline{AB}$ produced to E, $\angle ABC = 3\angle BAC$, $\angle ACB = 6\angle BAC$ and $\overline{AC}$ is parallel to BD then find $\angle DBE$ and $\angle CBD$.

(1) 54°, 108°
(2) 18°, 108°
(3) 54°, 100°
(4) 18°, 54°

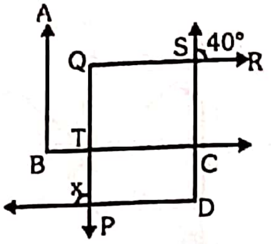
41.



In the above figure (not to scale), GF||BC, AB||PQ and AC||PR. If $\angle x = 40°$ and $\angle y = 110°$, then find $\angle QPR$.

(1) 70°
(2) 80°
(3) 60°
(4) None of the above

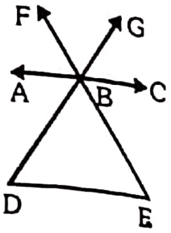
42.



In the above figure (not to scale), $\overline{AB} \parallel \overline{QP} \parallel \overline{SD}$ and also $\overline{QR} \parallel \overline{DP}$. Find x.

(1) 40° (2) 140° (3) 100° (4) None of the above

43. In the given figure, ∠BDE = x°, ∠FBG = (x + 2)° and ∠BED = (x + 7)°. Find the value of x.



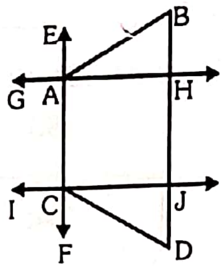
(1) 58

(2) 57

(3) 60

(4) 61

44.



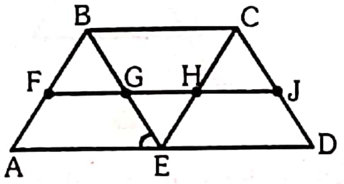
In the above figure, GH || IJ, AC || BD, AB and CD are bisectors of ∠EAH and ∠FCJ. Find ∠ABD + ∠BDC if ∠BAC = 3∠BDC.

(1) 80° (2) 90° (3) 100° (4) 110°

45. The supplement of an angle and the complement of another have the sum equal to half of a complete angle. If the greater angle is 10° more than the smaller, find the smaller angle.

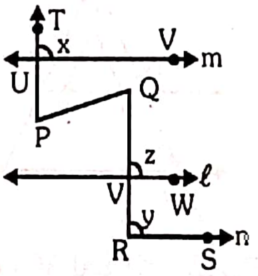
(1) 40° (2) 35° (3) 45° (4) 30°

46. In the figure below $\overline{FJ} \parallel \overline{AD}$, $\overline{BE} \parallel \overline{CD}$, ∠BGH = 130° and ∠CHG = 120°. If $\overline{AB} \parallel \overline{CE}$, then find ∠A + ∠D.



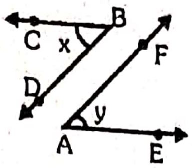
(1) 130° (2) 120° (3) 110° (4) 100°

47. In the figure below, m || ℓ || n and $\overline{PT} \parallel \overline{QR}$. If ∠TUV = x, ∠QRS = y and ∠QVW = z, then which o the following is necessarily true ?



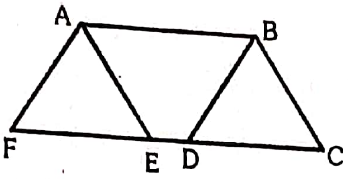
(1) x > y = z (2) x < y = z (3) x = y = z (4) Cannot be determined

48. In the figure below, $\overline{BC} \parallel \overline{AE}$ and $\overline{AF} \parallel \overline{BD}$. If $\angle CBD = x$ and $\angle FAE = y$, then find $|x - y|$.



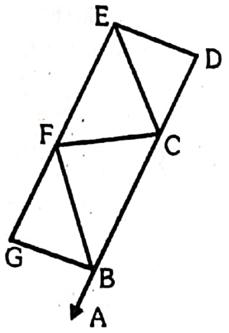
(1) 10° (2) 20° (3) Cannot be determined (4) None of the above

49. In the figure below $\overline{AB} \parallel \overline{FC}$, $\overline{AE} \parallel \overline{BC}$ and $\overline{AF} \parallel \overline{BD}$. If $\angle F = x$, $\angle C = y$, $\angle EAB = k$ and $\angle ABD = p$, then which of the following is correct?



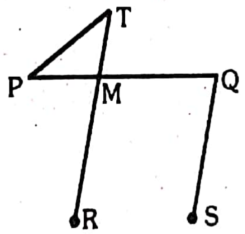
(1) $x = k$ and $y = p$ (2) $x = p$ and $y = k$ (3) Both (1) and (2) (4) None of the above

50. In the figure below, $AD \parallel \overline{GE}$, $\overline{GB} \parallel \overline{FC} \parallel \overline{ED}$ and $\overline{BF} \parallel \overline{CE}$ (not to scale). If $\angle ABG = x$, $\angle GBF = y$, $\angle CED = p$ and $\angle CDE = q$, then $x - y$ is



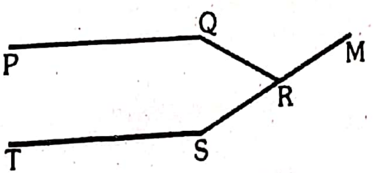
(1) $p - q$ (2) $q - p$ (3) Cannot be determined (4) None of these

51. In the figure below, $\overline{RT} \parallel \overline{SQ}$ (not to scale), $\angle Q = 80°$ and $\angle T = 30°$, find $\angle P$.



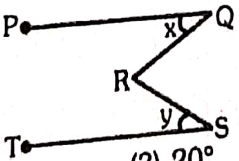
(1) 30° (2) 110° (3) 50° (4) None of the above

52. In the figure below, $\overline{PQ} \parallel \overline{TS}$, $\angle Q = 130°$ and $\angle S = 150°$. Find the measure of $\angle QRM$.



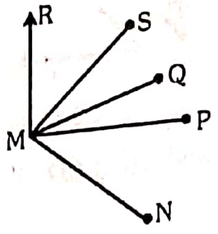
(1) 80° (2) 90° (3) 100° (4) 120°

53. In the figure below (not to scale), $\overline{PQ} \parallel \overline{TS}$, reflex $\angle QRS = 300°$ and $x - y = 30°$. Find the measure of y.



(1) 25° (2) 15° (3) 20° (4) 30°

54. In the figure below (not to scale), $\overline{MR} \perp \overline{MP}$, $\overline{MQ} \perp \overline{MN}$, and $\overline{MS}$ is bisector of ∠RMQ. If ∠PMN = 50°, then find the measure of ∠RMS.



(1) 25°
(2) 20°
(3) 30°
(4) 35°

55. If a, b are complementary angles and b, c are supplementary angles. If the average of a and c is 85°, then find a + b + c.

(1) 200°
(2) 220°
(3) 210°
(4) 240°

ANSWERS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	3	3	1	2	2	1	4	3	4	1	2	2	1	2	4	1	1	1	1	4
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	3	3	4	2	3	4	2	4	2	1	2	3	2	4	4	1	4	2	3	2
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55					
Ans.	1	2	2	2	1	3	3	4	2	2	3	3	2	1	2					